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There is no spooky action-at-a-distance in quantum correlations: Resolution of the EPR nonlocality puzzle

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Abstract

The long-standing puzzle of the nonlocal EPR correlations is solved. We show that there is no spooky action-at-a-distance at work in establishing the quantum correlations. The correct quantum mechanical correlations arise for the case of entangled particles under strict locality obeyed by the hidden variable used and by the rules for outcomes of measurements. The crucial input is the correct rule specified by complex numbers, without generating negative probabilities. Our approach shows that the EPR paradox of simultaneous reality for incompatible physical variables arise from their restrictive definition of physical reality.

Sixty five years ago, the most significant paper questioning a fundamental aspect of quantum phenomena was written by Einstein, Podolsky and Rosen (EPR) [1]. They addressed the question whether the wavefunction represented a complete description of reality in quantum mechanics, and argued that it didn't. Bohr's reply to this paper [2] was not sufficient to resolve the fundamental issues raised by EPR. Decades later, Bohm rephrased the EPR problem [3] in terms of particles correlated in their spin and this helped enormously in analyzing the problem with clarity. John Bell analyzed the EPR problem in the early sixties and established the Bell's inequalities obeyed by any local hidden variable theory for the correlations of entangled particles [4]. Quantum mechanical correlations calculated using the entangled wavefunction and spin operators violate these inequalities. Experiments, the first of which was by Freedman and Clauser [5] and the most remarkable by A. Aspect and collaborators [6], have established beyond doubt that (at least I take it that way for the purpose of this discussion) there cannot be a viable local realistic hidden variable description of quantum mechanics [7]. Further, these results also have been interpreted as evidence for nonlocal influences in quantum measurements involving entangled particles. Since no instruction set carried by the particles from their source of origin (possibly with the addition of several local hidden variables) can manage to create the correct correlations observed in experiments, the only way out seems to be that measurement of an observable on one of the particles in an entangled pair seems to convey the result of this measurement instantaneously to the other particle resulting in the correct behaviour of the other particle during a measurement on the second particle. Of course, the no signalling theorems in this context prohibit any faster than light signalling using this feature. Nevertheless, we seem to be stuck with the puzzling nonlocality which is probably the deepest mystery in the behaviour of entangled systems. In the quantum mechanical terminology, the measurement of an observable on one of the particles collapses the entire wavefunction instantaneously and nonlocally and the second particle acquires a definite value for the same observable, consistent with the correlation determined by the relevant conservation law.

Apart from the disturbing aspect of accepting the concept of nonlocality without being able to understand its nature, there is serious conflict with the spirit of relativity. As soon as we bring in the concept of one measurement being influenced nonlocally by the other, there is the concept of causal connection, propagating outside the light cone. The concept of simultaneity becomes important since both measurements can be labelled by local times. So, if one measurement precede the other in one frame, one can always find a moving frame in which the converse it true, the second measurement preceding the first [8].

We resolve the quantum nonlocality puzzle by correctly identifying the relevant local variables and local rules in quantum measurements of the type discussed in the context of entangled systems. *The correct quantum correlations emerge under strict locality.* We reproduce all the statistical results of measurements on the entangled system. There is no nonlocal influence across space-like separated points. There is no problem with the spirit of relativity any more since everything is strictly local (one measurement does not influence the other in any way). This is not a local realistic theory, and therfore do not contradict any theorem.

Consider the breaking up of a correlated state as in the standard Bohm version of the EPR problem [3]. The two particle go off in opposite directions and are in space-like regions. Two observers make measurements on these particles individually at space like separated regions with time stamps such that these results can be correlated later through a classical channel. We assume that strict locality is valid. *Measurements performed in space-like separated regions do not influence each other's outcomes in any way.* This covers all forms of locality and separability.

We assign local rules for the outcome of a particular measurement on each of the two particles. We cannot assign any undetermined but definite spin directions for these particles till a measurement is made (no description in terms of mixtures of definite states is possible). We now assume the existence of an internal variable for each of these two particles. The correlation at source is encoded in the relative value, or the difference, of this internal variable for the two particles. For simplicity let us call this internal variable a "phase", ϕ. Note that it is not a dynamical phase evolving as the particle propagates. It is an internal variable whose difference (possibly zero) remains constant for the particles of the correlated pair. The value of ϕ can vary from particle to particle, but the relative phase between the two particles in all correlated pairs is constant. In effect, this is our local hidden variable, whose undetermined value is carried by the particles such that the relative value remains constant for the whole ensemble of pairs in the experiment. Consider ϕ as a reference for the particles to determine the angle of a polarizer or analyzer encountered on their way, *locally* (we use the terms polarizer and analyzer in a generic way. They could be Stern-Gerlach like analyzers for spin 1/2 particles). The first particle encounters analyzer #1 kept at an angle θ_1 with respect to some global direction. We denote this angle of the analyzer with reference to ϕ as θ. Similarly, the second particle which has the internal phase angle $\phi + \phi_o$, where ϕ_o is a constant, encounters the second analyzer oriented at angle θ_2 at another space-like separated point. Let the orientation of this analyzer with respect to the internal phase angle of the second particle is θ'. We have $\theta - \theta' = \theta_1 - \theta_2 + \phi_o$. (The constant ϕ_o characterizes the correlation.)

An experiment in which each particle is analyzed by orienting the analyzers at various angles θ_1 and θ_2 is considered next. At each location the result is dichotomic denoted by $(+)$ for transmission and $(-)$ for absorption of each particle, for any angle of orientation. We specify the *local rule for transmission as a complex number, whose square gives the probability of transmission.* The complex number associated with particle #1 is $C_1 = \frac{1}{\sqrt{2}}\exp(i\theta s)$ for measurements at analyzer #1, and for particle #2 is $C_2 = \frac{1}{\sqrt{2}}\exp(i\theta' s)$ at analyzer #2. In these expressions, the quantity s is the spin (in units of $\hbar$) of the particle, 1 for photons and $\frac{1}{2}$ for spin-$\frac{1}{2}$ particles. The locality assumption is strictly enforced since the two complex functions depend only on local variables and on an internal variable determined at source and then individually carried by the particles without any subsequent communication of any sort. The factor $\frac{1}{\sqrt{2}}$ originates in the fact that the number of possible outcomes of the each measurement is two, with equal probability. The probabilities for the outcomes of measurements at each end are now correctly reproduced, for any angle of orientation. These probabilities are $C_1C_1^* = C_2C_2^* = \frac{1}{2}$. The correlation function for joint measurements is

$$\begin{aligned} U(\theta,\theta') &= C_1C_2^* + C_1^*C_2 = \frac{1}{2}(e^{is(\theta-\theta')} + e^{-is(\theta-\theta')}) \\ &= \cos\{s(\theta-\theta')\} = \cos\{s(\theta_1-\theta_2) + s\phi_o\}. \end{aligned} \quad (1)$$

We rewrite this as $U(\theta_1,\theta_2,\phi_o)$ since all references to the individual values of the hidden variable ϕ has dropped out. The square of $U(\theta_1,\theta_2,\phi_o)$ is the probability for joint transmission of the two particles through the analyzers kept at angles θ_1 and θ_2. Next we show that this correlation function generated from local rules represented by complex numbers, involving the difference of a local hidden variable for the two particles, reproduces correct quantum mechanical correlations. Since we have used local rules for transmission represented by complex numbers, we achieve this without violating Bell's theorem. This is not a local *realistic* theory. The usual concept of 'reality' is inadequate for unmeasured quantum observables, and we use a wider concept represented by complex numbers. In return we get rid of the most disturbing aspect in EPR correlations, the spooky action-at-a-distance.

Let us consider for discussion, the case of a correlated state of photons breaking up into *orthogonal polarization states.* This means that if one photon is transmitted through an analyzer on one side, the other one will not transmitted for the same orientation of the analyzer on the other side. So, perfect anti-correlation is implied for $\theta_1 - \theta_2 = 0$. If transmission is denoted as $+1$, and absorption as -1, the classical correlation function $P(\mathbf{a}, \mathbf{b}) = \frac{1}{N} \sum (A_i B_i)$ satisfies $-1 \leq P(\mathbf{a}, \mathbf{b}) \leq 1$. Here $(\mathbf{a}, \mathbf{b})$ denotes the two directions along which the analyzers are oriented and A_i and B_i are the two valued results. The Bell correlation $P(\mathbf{a}, \mathbf{b})$ (different from the correlation function $U(\theta_1, \theta_2, \phi_o)$ we derived) denotes the average of the quantity (*number of detections in coincidence* $-$ *number of detections in anticoincidence*), where 'coincidence' denotes both particles showing same value for the measurement and 'anticoincidence' denotes those with opposite values. The Bell correlation calculated from quantum mechanics for this case is given by $-\cos(2(\ (\theta_1 - \theta_2))$. That is, if the analyzers are oriented at a relative angle of $\pi/2$, perfect correlation is obtained. When the relative angle is $\pi/4$, the quantum mechanical correlation defined in the Bell way is zero, since there are as many coincidences as anticoincidences.

The correlation function we derived give, for the case of the photons discussed above,

$$U(\theta_1, \theta_2, \phi_o) = \cos\{(\theta_1 - \theta_2) + \phi_o\} \tag{2}$$

We set $\phi_o = \pi/2$ for denoting the correlation of the two orthogonal photons at source . Then we get

$$\begin{aligned} U(\theta_1, \theta_2, \phi_o) &= \cos\{(\theta_1 - \theta_2) + \pi/2\} \\ &= -\sin(\theta_1 - \theta_2) \end{aligned} \tag{3}$$

This is the correlation function for *joint transmission* through the two analyzers oriented at a relative angle of $(\theta_1 - \theta_2)$. The probability for coincidence detection is

$$U^2(\theta_1, \theta_2, \phi_o) = \sin^2(\theta_1 - \theta_2) \tag{4}$$

Correspondingly, the probability for anticoincidence is $1 - \sin^2(\theta_1 - \theta_2)$. Since the average of the quantity (number of coincidences $-$ number of anticoincidences) $=$

$$U^2(\theta_1, \theta_2, \phi_o) - (1 - U^2(\theta_1, \theta_2, \phi_o)) = 2U^2(\theta_1, \theta_2, \phi_o) - 1, \tag{5}$$

the correspondence between $P(\mathbf{a},\mathbf{b})$ and $U(\theta_1,\theta_2,\phi_o)$ is given by the general expression,

$$P(\mathbf{a},\mathbf{b}) = 2U^2(\theta_1,\theta_2,\phi_o) - 1 \tag{6}$$

We get for the Bell correlation,

$$P(\mathbf{a},\mathbf{b}) = 2\sin^2(\theta_1-\theta_2) - 1 = -\cos(2((\theta_1-\theta_2)) \tag{7}$$

This agrees completely with the usual quantum mechanical prediction derived by applying the relevant spin operators on the correct entangled state of the two photons.

Another important example is the case of the singlet state breaking up into two spin 1/2 particles propagating in opposite directions to spatially separated regions. We set $\phi_o = \pi$. Then our correlation function is

$$\begin{aligned} U(\theta_1,\theta_2,\phi_o) &= \cos\{s(\theta_1-\theta_2)+s\phi_o\} \\ &= \cos\{\frac{1}{2}(\theta_1-\theta_2)+\pi/2\} \\ &= -\sin\frac{1}{2}(\theta_1-\theta_2) \end{aligned} \tag{8}$$

The probability for joint transmission through two Stern-Gerlach analyzers oriented at relative angle $\theta_1-\theta_2$ is

$$U^2(\theta_1,\theta_2,\phi_o) = \sin^2(\frac{1}{2}(\theta_1-\theta_2)) \tag{9}$$

For the case of the two particles of the singlet state,

$$\begin{aligned} 2U^2(\theta_1,\theta_2,\phi_o) - 1 &= 2\sin^2(\frac{1}{2}(\theta_1-\theta_2)) - 1 \\ &= -\cos(\theta_1-\theta_2). \end{aligned} \tag{10}$$

This is again exactly same as the correct Bell correlation $P(\mathbf{a},\mathbf{b})$ for the quantum mechanical predictions obtained from the singlet entangled wavefunction and the Pauli spin operators. Perfect correlation is obtained for oppositely oriented analyzers and perfect anticorrelation for similarly oriented analyzers. When the analyzers are orthogonal, the correlation is zero.

We have correctly reproduced the quantum mechanical correlation using only a local hidden variable and local rules. The correct quantum mechanical correlation emerges from combining two local complex functions. No averaging over distribution of the hidden variable is done. Single events consisting of two independent measurements at the two analyzers obey the correlation we derived, and the probability for joint detection is given by the square of the correlation function. *In particular if the two analyzers happen to be in the same orientation, perfect correlation is reproduced every time within the strict locality assumption.* It is important to note that we have not used any information on the internal variable ϕ even in terms of distributions. It may be considered as a hidden variable appearing in the measurement prescriptions only through a complex number and has the nature of the origin of a non-dynamical phase associated with the quantum system. In fact, such a variable is not an external input additional to what is already available in the quantum mechanical description, since the zero of the phase of a wavefunction is unobservable. The fact that we have used a hidden variable appearing through a complex number rule for the outcome of a measurement is the crucial departure from the standard local hidden variable theories.

The miracle that was impossible with local rule described by a real function becomes possible when described by a locally determined complex number. The nonlocality puzzle in the EPR correlations is solved. Strict locality including Einstein locality is valid. We have found an answer to the EPR query regarding the completeness of quantum description. *It seems clear that even after performing a measurement on one of the particles of an entangled pair, the companion particle cannot be ascribed a reality in the sense of Einstein.* The companion particle's quantum properties remain as unmeasured and as 'un-collapsed' as ever, though the result of a measurement if performed, in the same direction, can be predicted with absolute certainty. (I will argue in another paper [9] that wavefunction collapse in the sense of Copenhagen interpretation and realization of an outcome happens only during actually performed measurements and not as a consequence of a measurement on a subsystem of an entangled system. See also Ref. [10]).

I would like to stress that the solution presented here resolves the problem, pointed out by EPR, of simultaneous reality of noncommuting observables. In fact our solution denies any reality to an actually unmeasured system. This suggests that there are physical systems in nature that are beyond the scope of the intuitive definition of EPR reality, just as the Copenhagen school maintained. The approach we have taken here gives predictions for correlations which are exactly the same as that would be obtained from the quantum wavefunction and operators, without the apparent nonlocal influence of one measurement on the other. The nonlocality apparent in entanglement correlation in quantum mechanics is not an inherent feature, but a conclusion forced on us when using a restrictive definition of physical reality.

The same analysis works for particles entangled in other sets of variables like momentum and coordinate, and energy and time. The results follow from the fact that all these cases of two particle entanglement can be mapped on to the spin-$\frac{1}{2}$ singlet problem with dichotomic outcomes [11]. For example, an experiment in which the particles entangled in momentum and position are used, with double slits for each of the particles, a two photon interference pattern described by the correlation $\frac{1}{2}(1 + \cos k\alpha(x_1 - x_2))$ will be observed with 100% visibility. In this, x_1 and x_2 are the coordinates of the two detectors separated by a space-like interval. k is the wave vector and α is a scaling factor for the angle subtended by the two slits at the detectors, source etc. The result which is seemingly nonlocal is reproduced applying local rules as given in our prescription. In the two photon interference experiment, the photon which is being detected at one detector has no nonlocal influence on the photon detected at the other detector.

How does all this reflect on a theory of measurement? Our approach has the interesting implication that the internal variable allows some amount of hidden determinism in the quantum physics of correlations. For an experiment in which single photons are allowed to fall on a polarizer, for example, the probability for two photons with the same internal variable showing the same outcome is unity. This is despite the fact that we are not able to predict the outcome in the case of one photon encountering a polarizer. This new feature requires further study. One may make a comment as to what happens to the internal variable after a measurement. The internal variable changes its value to $s\theta$, where θ is the angle of the analyzer. Subsequent measurement at another analyzer at angle θ' is governed by the local rule $Real(\exp[is(\theta - \theta')])$. Here, we have made the distinction between an unmeasured quantum system and a measured quantum system in specifying the local rule. This correctly reproduces average results of experiments done with multiple analyzers on single photons.

In summary, we have resolved the long standing and seriously disturbing issue of quantum nonlocality in the EPR problem. There is no nonlocal influence between correlated particles separated into space-like regions. The solution has important physical and philosophical implications regarding the nature of reality in quantum systems. Our approach shows that the EPR paradox of simultaneous reality for noncommuting physical variables arise from their restrictive definition of physical reality. The local hidden variable used in our discussion is a freedom available in the quantum theory itself, namely the undetermined zero of a non-dynamical phase or an internal variable. It appears in the theory only through rules represented by complex numbers for the outcomes in experiments, and only as a difference of values for the two correlated particles.

By restoring locality into the quantum measurements of entangled system we have realized one of Einstein's deepest wishes. But we have also shattered one of his dear dreams of a tangible concept of reality of unmeasured quantum systems. We could try to do better, but I have my doubts.

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